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ACTION OF A PRESSURE PULSE ON A CAVITY IN A VISCOUS LIQUID

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The case of the collapse of a cavity under the action of a constant external pressure p_0 was analyzed in [1]. There is a class of problems, however, in which the external action consists of brief pressure pulses. Such a situation occurs, for example, in the impact loading of porous solids.

Suppose that there is an empty spherical cavity of radius r_0 in a viscous incompressible liquid with a density ρ. The pressure $p_\infty(t, \tau)$ at infinity (far from the cavity) is an arbitrary function of time at $0 \le t \le \tau$ and is reduced to zero at $t > \tau$.

The motion is spherically symmetric and the Navier–Stokes equations describing it have the form

$$\frac{\partial u}{\partial r} + 2\frac{u}{r} = 0, \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \tag{1}$$

where u(r, t) is the velocity; p(r, t) is the pressure.

At the surface of the cavity a normal stress σ_{rr} is absent (the cavity is empty), and since $\sigma_{rr} = -p + 2\eta du/dr$, we have $p_1 = 2\eta(\partial u/\partial r)_1$. Here and later the values of quantities at the boundary are marked by the index 1; η is the coefficient of dynamic viscosity.

The second boundary condition will be

$$p = p_\infty(t, \tau) \quad \text{at} \quad r = \infty.$$

From the first equation of (1) we obtain $u(r, t) = u_1 r_1^2/r^2$.

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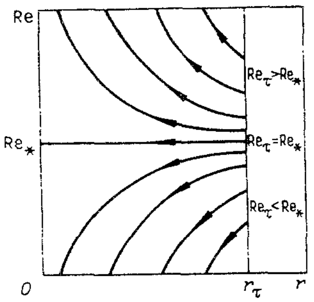

Fig. 1

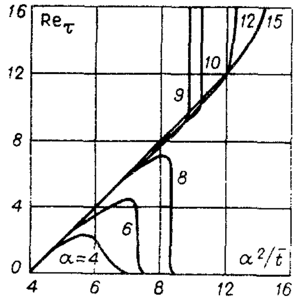

Fig. 2

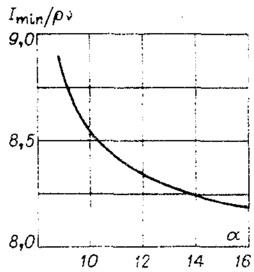

Fig. 3

Substituting this expression for u into the second equation of (1) and integrating from r_1 to ∞ with allowance for the boundary conditions for p, we obtain

$$\frac{du_1}{dr_1}+\frac{3}{2}\frac{u_1}{r_1}+\frac{p_\infty}{\rho u_1 r_1}+\frac{4\nu}{r_1^2}=0. \tag{2}$$

Let us consider the motion of the cavity at $t > \tau$, when $p_\infty = 0$. As the initial data for the radius of the cavity and the velocity of its surface, we take the values of these quantities at the time the action of the external pressure ceased. The solution has the form

$$u_1=\frac{\nu\left(\sqrt{\frac{r_\tau}{r_1}}(8-\mathrm{Re}_\tau)-8\right)}{r_1}, \tag{3}$$

where $\mathrm{Re}_\tau = |u_\tau| r_\tau/\nu$ is the value of the Reynolds number at $t = \tau$; u_τ and r_τ are the radius of the cavity and the velocity of its surface, respectively, at $t = \tau$.

Equation (3) allows one to obtain the law of variation of the Reynolds number at $t > \tau$

$$\mathrm{Re}=\sqrt{\frac{r_\tau}{r_1}}(\mathrm{Re}_\tau-8)+8, \tag{4}$$

from which it follows that when $\mathrm{Re}_\tau > 8$ the Reynolds number grows with a decrease in r_1 and approaches infinity as $r_1^{-1/2}$. When $\mathrm{Re}_\tau < 8$ the Reynolds number declines with a decrease in r_1 and becomes equal to zero at $r_1/r_\tau = (1-\mathrm{Re}_\tau/8)^2$. The velocity of the cavity boundary is reduced to zero at this value of the radius and its further motion ceases. When $\mathrm{Re}_\tau = 8$ the Reynolds number remains constant up to the complete collapse of the cavity.

Thus, the value of $\mathrm{Re}_\tau = 8$ is critical (we designate it as Re_*); it marks the boundary of the two different modes of collapse of the cavity (Fig. 1).

If the external pressure pulse is such that $\mathrm{Re}_\tau > \mathrm{Re}_*$, then collapse of the cavity occurs, with $u_1 \sim r_1^{-3/2}$ at small r_1. When $\mathrm{Re}_\tau = \mathrm{Re}_*$ collapse of the cavity also occurs, but $u_1 \sim r_1^{-1/2}$. When $\mathrm{Re}_\tau < \mathrm{Re}_*$ partial collapse of the cavity occurs.

An expression for the limiting radius of the cavity at any values of Re_τ can be obtained from (4):

$$\frac{r_{\lim}}{r_\tau}=\left(1-\frac{\mathrm{Re}_\tau}{\mathrm{Re}_*}\right)^2 U_-(\mathrm{Re}_*-\mathrm{Re}_\tau),$$

where $r_{\lim}$ is the limiting radius of the cavity; $U_-(x)$ is a unit antisymmetric function.

It follows from (3) that the realization of one or another mode of collapse of the cavity is determined by the value of Re_τ.

To determine the dependence of Re_τ on the pressure pulse $I=\int_0^\tau p\,dt$ we write the equation for the Reynolds number as a function of time, which follows from Eq. (2):

$$\frac{1}{\nu}\frac{d\,\mathrm{Re}}{dt}=\frac{1}{2}\frac{\mathrm{Re}^2}{r_1^2}+\frac{p_\infty}{\nu^2\rho}-4\frac{\mathrm{Re}}{r_1^2}. \tag{5}$$

Changing to the dimensionless quantities $\bar{t} = \nu t/r_0^2$, $\bar{r} = r_1/r_0$, and $\alpha = \sqrt{p_0/\rho}(r_0/\nu)$ and supplementing Eq. (5) with the equation of motion, we obtain the system

$$\frac{d\,\mathrm{Re}}{d\bar{t}} = \frac{\mathrm{Re}}{2\bar{r}^2}(\mathrm{Re} - 8) + \alpha^2, \quad \frac{d\bar{r}}{d\bar{t}} = -\frac{\mathrm{Re}}{\bar{r}} \tag{6}$$

with the initial conditions Re (0) = 0 and $\bar{r}(0) = 1$. In the case of an arbitrary dependence $p_\infty(t, \tau)$ one determines Re_τ numerically.

Let us consider the limiting case when the pressure pulse retains finite as $\tau \to 0$.

The solution of the system (6) has the form

$$\mathrm{Re}_\tau = I/\rho\nu, \ \bar{r} = 1.$$

For complete collapse of the cavity it is necessary that $I \geq \mathrm{Re}_* \rho\nu$.

Partial collapse of the cavity occurs when $I < \mathrm{Re}_* \rho\nu$, where

$$r_{\lim}/r_0 = (1 - I/\mathrm{Re}_*\rho\nu)^2.$$

For rectangular pressure pulses

$$p_\infty(t, \tau) = p_0 U\_(\tau - t), \ p_0 = \mathrm{const}$$

the quantity Re_τ essentially depends on the parameter $\alpha = \sqrt{p_0/\rho}(r_0/\nu)$: it either grows without limit with an increase in τ or, reaching a maximum at some τ, subsequently approaches zero. The motion of a cavity in a viscous liquid under the action of a constant pressure was studied in [1] and a critical value of $\alpha_* = 8.4$ was obtained for the parameter. When $\alpha > 8.4$ the velocity of the cavity boundary grows without limit as $r^{-3/2}$ with a decrease in radius, and therefore the Reynolds number also grows without limit: $\mathrm{Re} = |u_1| r_1/\nu \sim r_1^{-1/2}$. From the law (2) of variation in the velocity in the absence of external pressure it follows that when $\alpha < 8.4$ the maximum value is $\mathrm{Re}_\tau < \mathrm{Re}_*$ and partial collapse of the cavity occurs at any finite values of the pressure pulse. The dependence of Re_τ on the pressure pulse $I = p_0\tau$, obtained through numerical integration of the system (6), is presented in Fig. 2.

For $\alpha > 8.4$ there is a minimum value of the pressure pulse $I_{\min}$ at which the cavity collapses, in which case $\mathrm{Re}_\tau = \mathrm{Re}_*$. The dependence of $I_{\min}$ on α is presented in Fig. 3.

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DIMENSIONLESS EQUATIONS OF STATE AND ATTENUATION OF SHOCK WAVES

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1. Dimensionless Hugoniot Equations of State

Basic information as regards the compressibility of materials and their thermodynamics at high pressures is at present obtained from shock-wave experiments [1]. By using the wave velocity D and the mass velocity U, the pressures (as well as densities and specific energies) are made constant in them for determining the path of the Hugoniot adiabat. The remarkable empirical relation found in a number of experiments consists in that for many materials a linear dependence is observed between the shock-wave velocity and the downstream velocity of the matter, $D = C_0 + \lambda U$. This relation, together with the conservation laws, yields straightforward expressions for the shock pressure P_H, for the increase of the inner energy $E_H - E_0$, and for the deformation X:

$$X = 1 - \rho_0/\rho = U/(C_0 + \lambda U), \quad P_H = \rho_0 C_0^2 X/(1 - \lambda X)^2, \\ E_H - E_0 = 0.5 C_0^2 X^2/(1 - \lambda X)^2, \tag{1.1}$$

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